

## **A Sudden Movement across an Inclined Surface Breaking Strike-Slip Fault in an Elastic Layer Overlying a Viscoelastic Layer and a Viscoelastic Half-Space**

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**Abstract:** A multi-layered model is considered to represent the earth's lithosphere-asthenosphere system consisting of two parallel layers, first one is elastic and the second one is viscoelastic, overlying a viscoelastic half-space. They are assumed to be in welded contact. An inclined surface breaking strike-slip fault is taken to be situated in the first layer. Stresses are assumed to be accumulated in the lithosphere-asthenosphere system near the fault, due to some tectonic reason such as mantle convection. Analytical expressions for displacements, stresses and strains are obtained for both before and after the fault movement, for both the layers and the half-space. The surface shear strain, accumulation and release of shear stress near the fault and contour map for stress accumulation in the first layer for different inclination of the fault have been computed by using suitable mathematical techniques including integral transforms, and Green's functions, corresponding principle and MATLAB.

**Keywords:** Aseismic state, Green's function, Stress accumulation and release, Strike-slip fault, Viscoelastic.

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### **I. Introduction**

From the observational results it is found that the earthquake occurs in cyclic order. Two consecutive seismic events are generally separated by a long aseismic period. During this aseismic period there is no seismic disturbance in seismically active regions but slow ground deformations are observed. For better understanding of earthquake processes it is necessary to study the ground deformation during aseismic periods. Such study may be useful for developing the suitable mathematical model for developing the earthquake prediction program. In this paper we develop a theoretical model of lithosphere-asthenosphere system represented by a multi-layered half-space (elastic\ viscoelastic). A wide range of theoretical models has been developed by many authors such as Maruyama [1], Rybicki [2,3], Mukhopadhyay [4,5], Debnath and Sen [6,7,8,9], Debnath and Sen [10,11,12], Mondal and Sen [13]. They represent the lithosphere-asthenosphere system either by an elastic/ viscoelastic half-space or by a layered viscoelastic half-space.

### **II. Formulation**

We consider a theoretical model of lithosphere-asthenosphere system consisting of two parallel layers, the first one is elastic and the second one is viscoelastic, overlying a viscoelastic half-space. The material of viscoelastic layer and half-space are of Maxwell type. The depth of the elastic layer is  $h_1$  and that of viscoelastic layer is  $h_2$ , below the free surface. These layers and half-space are supposed to be in welded contact. A plane surface breaking strike-slip fault  $F$  with an inclination  $\theta$  with the horizontal is taken to be situated in the elastic layer. The length of  $F$  is assumed to be large compare to its width  $l$ . The upper and lower edges of the fault are horizontal.

We introduce a rectangular Cartesian co-ordinate system  $(y_1, y_2, y_3)$  with the plane free surface as the plane  $y_3 = 0$ ,  $y_3$ - axis is pointing into the layers and half-space,  $y_1$ - axis is taken along the strike of the fault on the free surface. For convenience of analysis we introduce another set of co-ordinate axes  $(y'_1, y'_2, y'_3)$  where  $y'_1$ - axis is taken along the upper edge of the fault and plane of the fault is denoted by  $y'_2 = 0$ , so that the fault is given by  $F: (y'_2 = 0, 0 \leq y'_3 \leq l)$ . The thickness  $h_1$  of the elastic layer is taken to be greater than the width  $l$  of the fault. With these choice of axes the elastic layer occupies the region  $0 \leq y_3 \leq h_1$ , the viscoelastic layer occupies the region  $h_1 \leq y_3 \leq h_2$  and the viscoelastic half-space is given by  $y_3 \geq h_2$ . The plane boundaries between the two layers and viscoelastic layer and half-space are given by  $y_3 = h_1$  and  $y_3 = h_2$  respectively. The relations between two co-ordinate system are given by

$$\left. \begin{aligned} y_1 &= y_1' \\ y_2 &= y_2' \sin \theta + y_3' \cos \theta \\ y_3 &= -y_2' \cos \theta + y_3' \sin \theta \end{aligned} \right\} \quad (1)$$

**Fig. 1** shows the section of the theoretical model by the plane  $y_1 = 0$ .

We assume that the length of the fault is large compared to its depth, so that the displacements, stresses and strains are independent of  $y_1$  and dependent on  $y_2, y_3$  and time  $t$ . Then the components of displacements, stresses and strain can be divided into two groups, one associated with strike-slip movement and another associated with dip-slip movement of the fault. Since in this model the strike-slip movement of the fault is considered, then the relevant components of displacement, stress and strain associated with this strike-slip movement are  $u_1, (\tau_{12}, \tau_{13}), (e_{12}, e_{13})$  for elastic layer,  $u_1', (\tau_{12}', \tau_{13}'), (e_{12}', e_{13}')$  for viscoelastic layer and  $u_1'', (\tau_{12}'', \tau_{13}''), (e_{12}'', e_{13}'')$  for viscoelastic half-space respectively.

## 2.1 Constitutive equations

### 2.1.1 Stress - strain relations

For the elastic layer, the stress-strain relation can be written in the following form :

$$\left. \begin{aligned} \tau_{12} &= \mu_1 \frac{\partial u_1}{\partial y_2} \\ \tau_{13} &= \mu_1 \frac{\partial u_1}{\partial y_3} \end{aligned} \right\} \quad (2)$$

$$\text{for } 0 \leq y_3 \leq h_1, -\infty < y_2 < \infty$$

where  $\mu_1$  is the rigidity of the elastic layer.

For the viscoelastic layer, the stress-strain relation can be written in the following form :

$$\left. \begin{aligned} \left( \frac{1}{\eta_2} + \frac{1}{\mu_2} \frac{\partial}{\partial t} \right) \tau_{12}' &= \frac{\partial^2 u_1'}{\partial t \partial y_2} \\ \left( \frac{1}{\eta_2} + \frac{1}{\mu_2} \frac{\partial}{\partial t} \right) \tau_{13}' &= \frac{\partial^2 u_1'}{\partial t \partial y_3} \end{aligned} \right\} \quad (3)$$

$$\text{for } h_1 \leq y_3 \leq h_2, t \geq 0, -\infty < y_2 < \infty$$

where  $\mu_2$  and  $\eta_2$  is the effective rigidity and effective viscosity respectively for viscoelastic layer.

For the viscoelastic half-space, the stress-strain relation can be written in the following form:

$$\left. \begin{aligned} \left( \frac{1}{\eta_3} + \frac{1}{\mu_3} \frac{\partial}{\partial t} \right) \tau_{12}'' &= \frac{\partial^2 u_1''}{\partial t \partial y_2} \\ \left( \frac{1}{\eta_3} + \frac{1}{\mu_3} \frac{\partial}{\partial t} \right) \tau_{13}'' &= \frac{\partial^2 u_1''}{\partial t \partial y_3} \end{aligned} \right\} \quad (4)$$

$$\text{for } y_3 \geq h_2, t \geq 0, -\infty < y_2 < \infty$$

where  $\mu_3$  and  $\eta_3$  is the effective rigidity and effective viscosity respectively for viscoelastic half-space.

### 2.1.2 Stress equations of motion

For a slow, aseismic, quasi-static deformation the magnitude of the inertial terms are very small compared to the other terms in the stress equation of motion and they can be neglected. Hence relevant stress satisfy the relations

$$\frac{\partial \tau_{12}}{\partial y_2} + \frac{\partial \tau_{13}}{\partial y_3} = 0 \quad (5)$$

$$\text{for } 0 \leq y_3 \leq h_1, -\infty < y_2 < \infty$$

$$\frac{\partial \tau_{12}'}{\partial y_2} + \frac{\partial \tau_{13}'}{\partial y_3} = 0 \quad (6)$$

$$\text{for } h_1 \leq y_3 \leq h_2, -\infty < y_2 < \infty$$

$$\frac{\partial \tau_{12}''}{\partial y_2} + \frac{\partial \tau_{13}''}{\partial y_3} = 0 \quad (7)$$

$$\text{for } y_3 \geq h_2, -\infty < y_2 < \infty$$

From (2)-(7) we get

$$\nabla^2 u_1 = 0 \quad (8)$$

$$\text{for } 0 \leq y_3 \leq h_1, -\infty < y_2 < \infty$$

$$\nabla^2 u_1' = 0 \quad (9)$$

$$\text{for } h_1 \leq y_3 \leq h_2, -\infty < y_2 < \infty$$

$$\nabla^2 u_1'' = 0 \quad (10)$$

$$\text{for } y_3 \geq h_2, -\infty < y_2 < \infty$$

### 2.2 Boundary conditions

Since the free surface is stress free and the layers are in welded contact then

$$\left. \begin{aligned} \tau_{13} &= 0 \text{ at } y_3 = 0 \\ \tau_{13} &= \tau'_{13} \text{ at } y_3 = h_1 \\ u_1 &= u'_1 \text{ at } y_3 = h_1 \\ \tau'_{13} &= \tau''_{13} \text{ at } y_3 = h_2 \\ u'_1 &= u''_1 \text{ at } y_3 = h_2 \\ \tau''_{13} &\rightarrow 0 \text{ as } y_3 \rightarrow \infty \\ \text{for } |y_2| &< \infty, t \geq 0 \end{aligned} \right\} \quad (11)$$

### 2.3 Condition at infinity

We assume that tectonic forces result in a shear strain far away from the fault which may change with time. Then we have the following boundary conditions

$$\left. \begin{aligned} e_{12} &\rightarrow (e_{12})_{0\infty} + g(t) \\ e'_{12} &\rightarrow (e'_{12})_{0\infty} + g(t) \\ e''_{12} &\rightarrow (e''_{12})_{0\infty} + g(t) \end{aligned} \right\} \quad (12)$$

where  $(e_{12})_{0\infty} = \lim_{|y_2| \rightarrow \infty} (e_{12})_0$ ,  $(e'_{12})_{0\infty} = \lim_{|y_2| \rightarrow \infty} (e'_{12})_0$ ,  $(e''_{12})_{0\infty} = \lim_{|y_2| \rightarrow \infty} (e''_{12})_0$ , where  $(e_{12})_0, (e'_{12})_0, (e''_{12})_0$  are the values of  $e_{12}, e'_{12}, e''_{12}$  at  $t = 0$  and  $g(t)$  is a continuous and slowly increasing function of  $t$  with  $g(0) = 0$ . Same  $g(t)$  is taken for layers and half-space, since they are in welded contact.

### 2.4 Initial Conditions

We measure the time  $t$  from a suitable instant when the model is in aseismic state and there is no seismic disturbance in it.  $(u_1)_0, (u'_1)_0, (u''_1)_0, (\tau_{12})_0, \dots, (e''_{12})_0$  are values of  $u_1, u'_1, \dots, e''_{12}$  at time  $t = 0$  and they satisfy the stress-strain relation (2)-(4), stress equation of motion (5)-(7), the equations (8)-(10) and boundary condition (11) and the condition at infinity (12) for respective medium.

## III. Displacements, Stresses and Strains in the Absence of Fault Movement

To obtain the solutions for displacements, stresses and strains in the absence of fault movement, we take Laplace transforms of the equation (2)-(12) with respect to time  $t$  and then the boundary value problem is converted to another boundary value problem in the transformed domain. Solving this boundary value problem we get the solution in transformed domain. Taking Laplace inverse transformation of the obtained results, we get the solution as:

$$\left. \begin{aligned} u_1 &= (u_1)_0 + y_2 g(t) \\ \tau_{12} &= (\tau_{12})_0 + \mu_1 g(t) \\ \tau_{13} &= (\tau_{13})_0 \\ e_{12} &= (e_{12})_0 + g(t) \\ \tau_{1'2'} &= \text{Stress components which tends to cause} \\ &\quad \text{strike - slip movement across the fault } F \\ &= \tau_{12} \sin \theta - \tau_{13} \cos \theta \\ &= (\tau_{1'2'})_0 + \mu_1 \sin \theta g(t) \end{aligned} \right\} \quad (13)$$

$$\text{for elastic layer } 0 \leq y_3 \leq h_1, -\infty < y_2 < \infty$$

and

$$\left. \begin{aligned} u'_1 &= (u'_1)_0 + y_2 g(t) \\ \tau'_{12} &= (\tau'_{12})_0 \exp\left(-\frac{\mu_2 t}{\eta_2}\right) + \mu_2 \int_0^t g_1(\tau) \exp\left\{-\frac{\mu_2(t-\tau)}{\eta_2}\right\} d\tau \\ \tau'_{13} &= (\tau'_{13})_0 \exp\left(-\frac{\mu_2 t}{\eta_2}\right) \end{aligned} \right\} \quad (14)$$

$$\text{for viscoelastic layer } h_1 \leq y_3 \leq h_2, -\infty < y_2 < \infty$$

$$\left. \begin{aligned} u_1'' &= (u_1'')_0 + y_2 g(t) \\ \tau_{12}'' &= (\tau_{12}'')_0 \exp\left(-\frac{\mu_3 t}{\eta_3}\right) + \mu_3 \int_0^t g_1(\tau) \exp\left\{-\frac{\mu_3(t-\tau)}{\eta_3}\right\} d\tau \\ \tau_{13}'' &= (\tau_{13}'')_0 \exp\left(-\frac{\mu_3 t}{\eta_3}\right) \end{aligned} \right\} (15)$$

for viscoelastic half – space  $y_3 \geq h_2, -\infty < y_2 < \infty$

where  $g_1(t) = \frac{d}{dt}\{g(t)\}$ . From (13), (14) and (15) we find that both  $\tau_{12}$  and  $\tau_{13}$  increase with time. When the accumulated stresses exceeds some threshold values the fault  $F$  undergoes sudden movement resulting in an earthquake.

#### IV. Displacements, stresses and strains after the restoration of aseismic state following a sudden strike-slip movement across the fault

It is to be noted that due to a sudden fault movement across the fault  $F$ , the accumulated stress will be released to some extent and the fault becomes locked again when the shear stress near the fault has sufficiently been released. The disturbance generated due to this sudden slip across the fault  $F$  will gradually die out within a short span of time. During this short period, the inertia terms cannot be neglected, so that our basic equations are no longer valid. We leave out this short span of time from our consideration and consider the model afresh from a suitable instant when the aseismic state re-established in the model. We determine the displacements, stresses and strains after the fault movement with respect to new time origin  $t = 0$ , so that all the equations (2)-(12) are also valid.

The sudden movement across  $F$  is characterized by a discontinuity in  $u_1$  and the discontinuity of  $u_1$  across  $F$  is defined as:

$$[u_1] = Uf(y_3') \text{ across } F (y_2' = 0, 0 \leq y_3' \leq l) \quad (16)$$

where  $[u_1] = \lim_{y_2' \rightarrow 0^+} u_1 - \lim_{y_2' \rightarrow 0^-} u_1$  and  $f(y_3')$  is a continuous function of  $y_3'$  and  $U$  is constant, independent of  $y_2'$  and  $y_3'$ . All the other components  $u_1', u_1'', \tau_{12}, \dots, e_{13}''$  are continuous everywhere in the model.

We try to obtain displacement and stresses for  $t \geq 0$  (i.e. with respect to new time origin) after the movement across  $F$  in the form

$$\left. \begin{aligned} u_1 &= (u_1)_1 + (u_1)_2 \\ \tau_{12} &= (\tau_{12})_1 + (\tau_{12})_2 \\ \tau_{13} &= (\tau_{13})_1 + (\tau_{13})_2 \\ e_{12} &= (e_{12})_1 + (e_{12})_2 \\ \\ u_1' &= (u_1')_1 + (u_1')_2 \\ \tau_{12}' &= (\tau_{12}')_1 + (\tau_{12}')_2 \\ \tau_{13}' &= (\tau_{13}')_1 + (\tau_{13}')_2 \\ \\ u_1'' &= (u_1'')_1 + (u_1'')_2 \\ \tau_{12}'' &= (\tau_{12}'')_1 + (\tau_{12}'')_2 \\ \tau_{13}'' &= (\tau_{13}'')_1 + (\tau_{13}'')_2 \end{aligned} \right\} (17)$$

where  $(u_1)_1, (\tau_{12})_1, \dots, (\tau_{13})_1$  satisfy all the equations (2)-(12) and continuous throughout the medium while  $(u_1)_2, (\tau_{12})_2, \dots, (\tau_{13})_2$  satisfy all the above relations (2)-(11) and also satisfy the dislocation condition (16) together with

$$\left. \begin{aligned} (e_{12})_2 &\rightarrow 0 \\ (e_{12}')_2 &\rightarrow 0 \\ (e_{12}'')_2 &\rightarrow 0 \end{aligned} \right\} \text{ as } |y_2| \rightarrow \infty, t \geq 0 \quad (18)$$

Then,

$$\left. \begin{aligned} (u_1)_1 &= (u_1)_p + y_2 g(t) \\ (\tau_{12})_1 &= (\tau_{12})_p + \mu_1 g(t) \\ (\tau_{13})_1 &= (\tau_{13})_p \\ (e_{12})_1 &= (e_{12})_p + g(t) \end{aligned} \right\} (19)$$

$$\left. \begin{aligned}
 (u_1)'_1 &= (u_1)'_p + y_2 g(t) \\
 (\tau_{12})_1 &= (\tau_{12})'_p \exp\left(-\frac{\mu_2 t}{\eta_2}\right) + \mu_2 \int_0^t g_1(\tau) \exp\left\{-\frac{\mu_2(t-\tau)}{\eta_2}\right\} d\tau \\
 (\tau_{13})_1 &= (\tau_{13})'_p \exp\left(-\frac{\mu_2 t}{\eta_2}\right)
 \end{aligned} \right\} (20)$$

$$\left. \begin{aligned}
 (u_1)''_1 &= (u_1)''_p + y_2 g(t) \\
 (\tau_{12})''_1 &= (\tau_{12})''_p \exp\left(-\frac{\mu_3 t}{\eta_3}\right) + \mu_3 \int_0^t g_1(\tau) \exp\left\{-\frac{\mu_3(t-\tau)}{\eta_3}\right\} d\tau \\
 (\tau_{13})''_1 &= (\tau_{13})''_p \exp\left(-\frac{\mu_3 t}{\eta_3}\right)
 \end{aligned} \right\} (21)$$

where  $(u_1)_p, (\tau_{12})_p, \dots, (\tau_{13})_p$  are the values of  $(u_1)_1, (\tau_{12})_1, \dots, (\tau_{13})_1$  respectively at  $t = 0$  (i.e. new time origin).

To obtain the solutions for  $(u_1)_2, (\tau_{12})_2, \dots, (\tau_{13})_2$  for  $t \geq 0$  we take Laplace transformation of the equations which satisfy the equations (2)-(11) and the dislocation condition (16) and condition (18), with respect to time  $t$ . The resulting boundary value problem involving  $(\bar{u}_1)_2, (\bar{\tau}_{12})_2, \dots, (\bar{\tau}_{13})_2$ , which are the Laplace transform of  $(u_1)_2, (\tau_{12})_2, \dots, (\tau_{13})_2$  respectively with respect to time  $t$ , can be solved by using a modified Green's function technique developed by Maruyama [1] and Rybicki [2,3] and correspondence principle. The Green's function for multilayered model are developed by Rybicki [3] and we use these results after some necessary simplification. On taking inverse Laplace transformation, we obtain the solutions for  $(u_1)_2, (\tau_{12})_2, \dots, (\tau_{13})_2$  for  $t \geq 0$ . The complete solutions for  $u_1, u_1', \dots, \tau_{13}$  are obtained as follows:

$$\left. \begin{aligned}
 u_1(y_2, y_3, t) &= (u_1)_p + y_2 g(t) + \frac{U}{2\pi} \psi_1(y_2, y_3, t) \\
 \tau_{12}(y_2, y_3, t) &= (\tau_{12})_p + \mu_1 g(t) + \frac{\mu_1 U}{2\pi} \psi_2(y_2, y_3, t) \\
 \tau_{13}(y_2, y_3, t) &= (\tau_{13})_p + \frac{\mu_1 U}{2\pi} \psi_3(y_2, y_3, t) \\
 e_{12}(y_2, y_3, t) &= (e_{12})_p + g(t) + \frac{U}{2\pi} \psi_2(y_2, y_3, t) \\
 \tau_{1'2'} &= (\tau_{1'2'})_p + \mu_1 g(t) \sin \theta + \frac{\mu_1 U}{2\pi} (\psi_2 \sin \theta - \psi_3 \cos \theta)
 \end{aligned} \right\} (22)$$

For elastic layer  $0 \leq y_3 \leq h_1, |y_2| < \infty$

$$\left. \begin{aligned}
 u_1'(y_2, y_3, t) &= (u_1)'_p + y_2 g(t) + \frac{U}{\pi} \phi_1(y_2, y_3, t) \\
 \tau_{12}'(y_2, y_3, t) &= (\tau_{12})'_p \exp\left(-\frac{\mu_2 t}{\eta_2}\right) + \\
 &\mu_2 \int_0^t g_1(\tau) \exp\left\{-\frac{\mu_2(t-\tau)}{\eta_2}\right\} d\tau + \frac{U}{\pi} \phi_2(y_2, y_3, t) \\
 \tau_{13}'(y_2, y_3, t) &= (\tau_{13})'_p \exp\left(-\frac{\mu_2 t}{\eta_2}\right) + \frac{U}{\pi} \phi_3(y_2, y_3, t)
 \end{aligned} \right\} (23)$$

For viscoelastic layer  $h_1 \leq y_3 \leq h_2, |y_2| < \infty$

$$\left. \begin{aligned} u_1''(y_2, y_3, t) &= (u_1'')_p + y_2 g(t) + \frac{2U}{\pi} \chi_1(y_2, y_3, t) \\ \tau_{12}''(y_2, y_3, t) &= (\tau_{12}'')_p \exp\left(-\frac{\mu_3 t}{\eta_3}\right) + \\ &\mu_3 \int_0^t g_1(\tau) \exp\left\{-\frac{\mu_3(t-\tau)}{\eta_3}\right\} d\tau + \frac{2U}{\pi} \chi_2(y_2, y_3, t) \\ \tau_{13}''(y_2, y_3, t) &= (\tau_{13}'')_p \exp\left(-\frac{\mu_3 t}{\eta_3}\right) + \frac{2U}{\pi} \chi_3(y_2, y_3, t) \end{aligned} \right\} (24)$$

For viscoelastic half – space  $y_3 \geq h_2, |y_2| < \infty$

Where  $\psi_1(y_2, y_3, t), \psi_2(y_2, y_3, t), \psi_3(y_2, y_3, t); \phi_1(y_2, y_3, t), \phi_2(y_2, y_3, t), \phi_3(y_2, y_3, t)$  and  $\chi_1(y_2, y_3, t), \chi_2(y_2, y_3, t), \chi_3(y_2, y_3, t)$  are given in Appendix.

For locked fault, analytical investigations show that the displacements, stresses and strains will be finite and single valued everywhere in the model including the tip of the fault  $F$  if the following sufficient conditions are satisfied by  $f(y_3')$

(i)  $f(y_3')$  and  $f'(y_3')$  are both continuous function of  $y_3'$  for  $0 \leq y_3' \leq l$ .

(ii)  $f(l) = 0$  and  $f'(0) = f'(l) = 0$ .

(iii) Either  $f''(y_3')$  is continuous in  $0 \leq y_3' \leq l$  or  $f''(y_3')$  is continuous in  $0 \leq y_3' \leq l$  except for a finite number of points of finite discontinuity in  $0 \leq y_3' \leq l$  or  $f''(y_3')$  is continuous in  $0 < y_3' < l$  except possibly for a finite number of points of finite discontinuity and for the end points of  $(0, l)$ , there exist real constant  $m, n$  both  $< 1$  such that  $(y_3')^m f''(y_3') \rightarrow 0$  or to a finite limit as  $y_3' \rightarrow 0 + 0$  and  $(l - y_3')^n f''(y_3') \rightarrow 0$  or to a finite as limit  $y_3' \rightarrow l - 0$ .

These conditions imply that the displacements, stresses and strains will be bounded everywhere in the model.

#### IV. Numerical Results And Discussion

To study the surface displacements, surface shear strain accumulation or release and the shear stress near fault tending to cause strike-slip movement, we choose  $f(y_3') = \frac{(y_3'^2 - l^2)^2}{l^4}$ , for which displacements, stresses and strains remain finite, everywhere in the model. The following values of the model parameters are taken for numerical computations :  $l = 10$  km. is the width of the fault  $F$ ,  $h_1 = 40$  km., and  $h_2 = 300$  km. from free surface, representing the upper part of the lithosphere and the upper part of the asthenosphere respectively.  $\mu_1 = 0.63 \times 10^{12}$  dyne/sq.cm.,  $\mu_2 = 0.75 \times 10^{12}$  dyne/sq.cm.,  $\mu_3 = 2.42 \times 10^{12}$  dyne/sq.cm.,  $\eta_2 = 3 \times 10^{21}$  poise,  $\eta_3 = 3.5 \times 10^{21}$  poise,  $U = 40$  cm., is the slip across the fault  $F$  and for the function  $g(t) = kt$ , we take  $k = 3.2 \times 10^{-14}$ . These values are taken from different books and research publications (e.g. Cathles[14], Clift [15], Karato [16], Bullen and Bolt [17]).

For numerical results we compute the following quantities:

(i) The residual surface shear strain due to fault movement near the fault one year after the time of restoration of aseismic state

$$\begin{aligned} E_{12} &= [e_{12} - (e_{12})_p - g(t)]_{y_3=0, t=1 \text{ year}} \\ &= \frac{U}{2\pi} \psi_2(y_2, y_3, t) \end{aligned}$$

(ii) Change in shear stress near fault  $F$  due to movement across  $F$ :

$$\begin{aligned} T_{1'2'} &= \tau_{1'2'} - (\tau_{1'2'})_p - \mu_1 g(t) \\ &= \frac{\mu_1 U}{2\pi} (\psi_2 \sin \theta - \psi_3 \cos \theta) \end{aligned}$$

The magnitude of the residual surface shear strain is found to be of the order of  $10^{-6}$  which is in conformity with the observed result. However the strain pattern depends upon the fault parameters. In Fig. 2 the strain curve is symmetric about the fault line for vertical fault and the maximum strain occurred at  $y_2 = 0$ . As the inclination of the fault with the horizontal increase then the value of  $y_2$  where the maximum strain occurred increases, e.g. when  $\theta = \frac{\pi}{3}$ , the maximum strain occurred at  $y_2 = 2$  km. and when  $\theta = \frac{\pi}{4}$ , the maximum strain occurred at  $y_2 = 3$  km.

In Fig. 3-5 the region of stress accumulation and release has been shown due to the fault movement across  $F$ , for different inclination  $\theta$ . The region marked by 'R' is the region of stress released and while region marked by 'A' is the region of stress accumulation. Thus if a second fault were situated in region 'R' a movement across  $F$  will result in reduction of stress near the second fault. Thus a possible movement across the

second fault will be delayed due to the movement across  $F$ . On the other hand if the second fault be situated in the region 'A', a movement across  $F$  will enhanced the rate of stress accumulation near the second fault and thereby a possible movement across the second fault will be advanced. If  $h_2 = 0$ , the Fig. 6, is quite different from the previous three figures. Fig. 6 represents the accumulation/ release region due to movement across the vertical fault  $F$  with the model in which viscoelastic layer is absent (i.e.  $h_2 = 0$ ). We may observed the difference of accumulation/ release region on comparing with Fig. 3,  $\theta = \frac{\pi}{2}$ , when the viscoelastic layer is present.

The contour map of shear stress for  $\theta = \frac{\pi}{2}$  and  $\theta = \frac{\pi}{3}$  has been shown in Fig. 7 and 8 respectively.

### V. Conclusions

To understand the mechanism of stress accumulation in a seismically active region, modeling of observed ground deformation during the aseismic period has important role. The rate of stress accumulation/ release under the action of the tectonic forces and the effect of fault movement on the nature of stress accumulation pattern during the aseismic period, (being the preparatory period for the next major seismic event), may give us more insight into the earthquake mechanism. Such studies are useful in formulation of an effective program of earthquake prediction.

### VI. Appendix

Displacements, stresses and strains after the restoration of aseismic state following a sudden strike-slip movement across the fault:

The displacements, stress and strains for  $t \geq 0$  with new time origin after restoration of aseismic state followed by a sudden movement have been found in the form given by (17) where  $(u_1)_1, (\tau_{12})_1, \dots, (\tau_{13})_1$  are given by (19)-(21) and  $(u_1)_2, (\tau_{12})_2, \dots, (\tau_{13})_2$  satisfy (2)-(11), (16) and (18). To obtain the solutions we take Laplace transformation of these relations with respect to  $t$  and we get

$$\left. \begin{aligned} (\bar{\tau}_{12})_2 &= \mu_1 \frac{\partial (\bar{u}_1)_2}{\partial y_2} \\ (\bar{\tau}_{13})_2 &= \mu_1 \frac{\partial (\bar{u}_1)_2}{\partial y_3} \end{aligned} \right\} \quad (25)$$

for  $0 \leq y_3 \leq h_1, -\infty < y_2 < \infty$

$$\left. \begin{aligned} (\bar{\tau}'_{12})_2 &= \bar{\mu}_2 \frac{\partial (\bar{u}'_1)_2}{\partial y_2} \\ (\bar{\tau}'_{13})_2 &= \bar{\mu}_2 \frac{\partial (\bar{u}'_1)_2}{\partial y_3} \end{aligned} \right\} \quad (26)$$

for  $h_1 \leq y_3 \leq h_2, -\infty < y_2 < \infty$

$$\left. \begin{aligned} (\bar{\tau}''_{12})_2 &= \bar{\mu}_3 \frac{\partial (\bar{u}''_1)_2}{\partial y_2} \\ (\bar{\tau}''_{13})_2 &= \bar{\mu}_3 \frac{\partial (\bar{u}''_1)_2}{\partial y_3} \end{aligned} \right\} \quad (27)$$

for  $y_3 \geq h_2, -\infty < y_2 < \infty$

where  $\bar{\mu}_2 = \frac{p}{\frac{p}{\mu_2} + 1}$  and  $\bar{\mu}_3 = \frac{p}{\frac{p}{\mu_3} + \frac{1}{\eta_3}}$

$$\frac{\partial (\bar{\tau}_{12})_2}{\partial y_2} + \frac{\partial (\bar{\tau}_{13})_2}{\partial y_3} = 0 \quad (28)$$

for  $0 \leq y_3 \leq h_1, -\infty < y_2 < \infty$

$$\frac{\partial (\bar{\tau}'_{12})_2}{\partial y_2} + \frac{\partial (\bar{\tau}'_{13})_2}{\partial y_3} = 0 \quad (29)$$

for  $h_1 \leq y_3 \leq h_2, -\infty < y_2 < \infty$

$$\frac{\partial (\bar{\tau}''_{12})_2}{\partial y_2} + \frac{\partial (\bar{\tau}''_{13})_2}{\partial y_3} = 0 \quad (30)$$

for  $y_3 \geq h_2, -\infty < y_2 < \infty$

$$\nabla^2 (\bar{u}_1)_2 = 0 \quad (31)$$

for  $0 \leq y_3 \leq h_1, -\infty < y_2 < \infty$

$$\nabla^2(\bar{u}'_1)_2 = 0 \quad (32)$$

$$\text{for } h_1 \leq y_3 \leq h_2, -\infty < y_2 < \infty$$

$$\nabla^2(\bar{u}''_1)_2 = 0 \quad (33)$$

$$\text{for } y_3 \geq h_2, -\infty < y_2 < \infty$$

$$\left. \begin{aligned} (\bar{\tau}_{13})_2 &= 0 \text{ at } y_3 = 0 \\ (\bar{\tau}_{13})_2 &= (\bar{\tau}'_{13})_2 \text{ at } y_3 = h_1 \\ (\bar{u}_1)_2 &= (\bar{u}'_1)_2 \text{ at } y_3 = h_1 \\ (\bar{\tau}'_{13})_2 &= (\bar{\tau}''_{13})_2 \text{ at } y_3 = h_2 \\ (\bar{u}'_1)_2 &= (\bar{u}''_1)_2 \text{ at } y_3 = h_2 \\ (\bar{\tau}_{13})_2 &\rightarrow 0 \text{ as } y_3 \rightarrow \infty \\ &\text{for } |y_2| < \infty \end{aligned} \right\} \quad (34)$$

$$\left. \begin{aligned} (\bar{e}_{12})_2 &\rightarrow 0 \\ (\bar{e}'_{12})_2 &\rightarrow 0 \\ (\bar{e}''_{12})_2 &\rightarrow 0 \\ &\text{as } |y_2| \rightarrow \infty \end{aligned} \right\} \quad (35)$$

and

$$[(\bar{u}_1)_2] = \frac{u}{p} f(y'_3) \text{ across } F: (y'_2 = 0, 0 \leq y'_3 \leq l) \quad (36)$$

where

$$\{(\bar{u}_1)_2, \dots, (\bar{\tau}_{13})_2\} = \int_0^\infty \{(u_1)_2, \dots, (\tau_{13})_2\} \exp(-pt) dt$$

and  $p$  being the Laplace transform variable.

The boundary value problem (25)-(36) can be solved by using modified form of Green's function technique developed by Maruyama [1] and Rybicki [3] with correspondence principle and following them we get

$$(\bar{u}_1)_2(Q_1, P) = \int_F [(\bar{u}_1)_2(P)] \{G_{12(1)}^1(Q_1, P) dx_3 - G_{13(1)}^1(Q_1, P) dx_2\} \quad (37)$$

$$(\bar{u}'_1)_2(Q_2, P) = \int_F [(\bar{u}_1)_2(P)] \{G_{12(2)}^1(Q_2, P) dx_3 - G_{13(2)}^1(Q_2, P) dx_2\} \quad (38)$$

$$(\bar{u}''_1)_2(Q_3, P) = \int_F [(\bar{u}_1)_2(P)] \{G_{12(3)}^1(Q_3, P) dx_3 - G_{13(3)}^1(Q_3, P) dx_2\} \quad (39)$$

where  $Q_1(y_1, y_2, y_3)$ ,  $Q_2(y_1, y_2, y_3)$ ,  $Q_3(y_1, y_2, y_3)$  are the field points in the first layer, second layer and half-space respectively and  $P(x_1, x_2, x_3)$  is any point on the fault  $F$  and  $[(u_1)_2(P)]$  is the magnitude of discontinuity of  $u_1$  across the fault  $F$  and

$$[(\bar{u}_1)_2(P)] = \frac{u}{p} f(x_3) \quad (40)$$

Suppose  $P(x'_1, x'_2, x'_3)$  is any point on  $F$  with respect to origin  $O'(0, 0, 0)$  and  $P(x_1, x_2, x_3)$  is any point on  $F$  with respect to the origin  $O$ , they are connected by the relations

$$\left. \begin{aligned} x_1 &= x'_1 \\ x_2 &= x'_2 \sin \theta + x'_3 \cos \theta \\ x_3 &= -x'_2 \cos \theta + x'_3 \sin \theta \end{aligned} \right\} \quad (41)$$

On  $F: x'_2 = 0, 0 \leq x'_3 \leq l$ . So  $dx'_2 = 0$ .

Therefore,

$$\left. \begin{aligned} x_1 &= x'_1 \\ x_2 &= x'_3 \cos \theta, dx_2 = \cos \theta dx'_3 \\ x_3 &= x'_3 \sin \theta, dx_3 = \sin \theta dx'_3 \end{aligned} \right\} \quad (42)$$

$$(\bar{u}_1)_2(Q_1, P) = \frac{U}{p} \int_0^l f(x'_3) \{G_{12(1)}^1(Q_1, P) \sin \theta - G_{13(1)}^1(Q_1, P) \cos \theta\} dx'_3 \quad (43)$$

$$(\bar{u}'_1)_2(Q_2, P) = \frac{U}{p} \int_0^l f(x'_3) \{G_{12(2)}^1(Q_2, P) \sin \theta - G_{13(2)}^1(Q_2, P) \cos \theta\} dx'_3 \quad (44)$$

$$(\bar{u}''_1)_2(Q_3, P) = \frac{U}{p} \int_0^l f(x'_3) \{G_{12(3)}^1(Q_3, P) \sin \theta - G_{13(3)}^1(Q_3, P) \cos \theta\} dx'_3 \quad (45)$$

Where

$$G_{12(1)}^1(Q_1, P) = \left. \int_0^\infty \frac{[A_1(\lambda)e^{-\lambda y_3} + B_1(\lambda)e^{\lambda y_3}] \sin[\lambda(x_2 - y_2)]}{2\pi(x_2 - y_2)^2 + (x_3 - y_3)^2} d\lambda \right\} \quad (46)$$

$$G_{13(1)}^1(Q_1, P) = \left. \int_0^\infty \frac{[C_1(\lambda)e^{-\lambda y_3} + D_1(\lambda)e^{\lambda y_3}] \cos[\lambda(x_2 - y_2)]}{2\pi(x_2 - y_2)^2 + (x_3 - y_3)^2} d\lambda \right\} \quad (47)$$

$$G_{12(2)}^1(Q_2, P) = \int_0^\infty [A_2(\lambda)e^{-\lambda y_3} + B_2(\lambda)e^{\lambda y_3}] \sin[\lambda(x_2 - y_2)] d\lambda \quad (48)$$

$$G_{13(2)}^1(Q_2, P) = \int_0^\infty [C_2(\lambda)e^{-\lambda y_3} + D_2(\lambda)e^{\lambda y_3}] \cos[\lambda(x_2 - y_2)] d\lambda \quad (49)$$

$$G_{12(3)}^1(Q_3, P) = \int_0^\infty [A_3(\lambda)e^{-\lambda y_3} + B_3(\lambda)e^{\lambda y_3}] \sin[\lambda(x_2 - y_2)] d\lambda \quad (50)$$

$$G_{13(3)}^1(Q_3, P) = \int_0^\infty [C_3(\lambda)e^{-\lambda y_3} + D_3(\lambda)e^{\lambda y_3}] \cos[\lambda(x_2 - y_2)] d\lambda \quad (51)$$

where

$$\left. \begin{aligned} A_1 &= -\frac{1}{2\pi\Delta_2} \left[ \begin{aligned} &(\bar{\gamma}_1 + 1)(\bar{\gamma}_2 + 1)e^{\lambda(2h_1+2h_2-x_3)} + \\ &(\bar{\gamma}_1 - 1)(\bar{\gamma}_2 - 1)e^{\lambda(4h_1-x_3)} - \\ &(\bar{\gamma}_1 + 1)(\bar{\gamma}_2 - 1)e^{\lambda(2h_1+x_3)} - \\ &(\bar{\gamma}_1 - 1)(\bar{\gamma}_2 + 1)e^{\lambda(2h_2+x_3)} \end{aligned} \right] \\ B_1 &= \frac{1}{2\pi\Delta_2} \left\{ \begin{aligned} &(\bar{\gamma}_1 + 1)(\bar{\gamma}_2 - 1)[e^{\lambda(2h_1+x_3)} + e^{\lambda(2h_1-x_3)}] + \\ &(\bar{\gamma}_1 - 1)(\bar{\gamma}_2 + 1)[e^{\lambda(2h_2+x_3)} + e^{\lambda(2h_2-x_3)}] \end{aligned} \right\} \\ A_2 &= -\frac{(\bar{\gamma}_2+1)}{\pi\Delta_2} [e^{\lambda(2h_1+2h_2+x_3)} + e^{\lambda(2h_1+2h_2-x_3)}] \\ B_2 &= \frac{(\bar{\gamma}_2-1)}{\pi\Delta_2} [e^{\lambda(2h_1+x_3)} + e^{\lambda(2h_1-x_3)}] \\ A_3 &= -\frac{2}{\pi\Delta_2} [e^{\lambda(2h_1+2h_2+x_3)} + e^{\lambda(2h_1+2h_2-x_3)}] \end{aligned} \right\} \quad (52)$$

Continued equation (52)

$$\left. \begin{aligned} C_1 &= -\frac{1}{2\pi\Delta_2} \left[ \begin{aligned} &(\bar{\gamma}_1 + 1)(\bar{\gamma}_2 + 1)e^{\lambda(2h_1+2h_2-x_3)} + \\ &(\bar{\gamma}_1 - 1)(\bar{\gamma}_2 - 1)e^{\lambda(4h_1-x_3)} + \\ &(\bar{\gamma}_1 + 1)(\bar{\gamma}_2 - 1)e^{\lambda(2h_1+x_3)} + \\ &(\bar{\gamma}_1 - 1)(\bar{\gamma}_2 + 1)e^{\lambda(2h_2+x_3)} \end{aligned} \right] \\ D_1 &= -\frac{1}{2\pi\Delta_2} \left\{ \begin{aligned} &(\bar{\gamma}_1 + 1)(\bar{\gamma}_2 - 1)[e^{\lambda(2h_1+x_3)} - e^{\lambda(2h_1-x_3)}] + \\ &(\bar{\gamma}_1 - 1)(\bar{\gamma}_2 + 1)[e^{\lambda(2h_2+x_3)} - e^{\lambda(2h_2-x_3)}] \end{aligned} \right\} \\ C_2 &= \frac{(\bar{\gamma}_2 + 1)}{\pi\Delta_2} [e^{\lambda(2h_1+2h_2+x_3)} - e^{\lambda(2h_1+2h_2-x_3)}] \\ D_2 &= \frac{(\bar{\gamma}_2 - 1)}{\pi\Delta_2} [e^{\lambda(2h_1-x_3)} - e^{\lambda(2h_1+x_3)}] \\ C_3 &= \frac{2}{\pi\Delta_2} [e^{\lambda(2h_1+2h_2+x_3)} - e^{\lambda(2h_1+2h_2-x_3)}] \end{aligned} \right\}$$

Along the fault  $x_2 = x'_3 \cos \theta$  and  $x_3 = x'_3 \sin \theta$ . So the equation (52) becomes

$$\left. \begin{aligned} A_1 &= -\frac{1}{2\pi\Delta_2} \left[ \begin{aligned} &(\bar{\gamma}_1 + 1)(\bar{\gamma}_2 + 1)e^{\lambda(2h_1+2h_2-x'_3 \sin \theta)} + \\ &(\bar{\gamma}_1 - 1)(\bar{\gamma}_2 - 1)e^{\lambda(4h_1-x'_3 \sin \theta)} - \\ &(\bar{\gamma}_1 + 1)(\bar{\gamma}_2 - 1)e^{\lambda(2h_1+x'_3 \sin \theta)} - \\ &(\bar{\gamma}_1 - 1)(\bar{\gamma}_2 + 1)e^{\lambda(2h_2+x'_3 \sin \theta)} \end{aligned} \right] \\ B_1 &= \frac{1}{2\pi\Delta_2} \left\{ \begin{aligned} &(\bar{\gamma}_1 + 1)(\bar{\gamma}_2 - 1) [e^{\lambda(2h_1+x'_3 \sin \theta)} + e^{\lambda(2h_1-x'_3 \sin \theta)}] + \\ &(\bar{\gamma}_1 - 1)(\bar{\gamma}_2 + 1) [e^{\lambda(2h_2+x'_3 \sin \theta)} + e^{\lambda(2h_2-x'_3 \sin \theta)}] \end{aligned} \right\} \\ A_2 &= -\frac{(\bar{\gamma}_2+1)}{\pi\Delta_2} [e^{\lambda(2h_1+2h_2+x'_3 \sin \theta)} + e^{\lambda(2h_1+2h_2-x'_3 \sin \theta)}] \\ B_2 &= \frac{(\bar{\gamma}_2-1)}{\pi\Delta_2} [e^{\lambda(2h_1+x'_3 \sin \theta)} + e^{\lambda(2h_1-x'_3 \sin \theta)}] \\ A_3 &= -\frac{2}{\pi\Delta_2} [e^{\lambda(2h_1+2h_2+x'_3 \sin \theta)} + e^{\lambda(2h_1+2h_2-x'_3 \sin \theta)}] \end{aligned} \right\} \quad (53)$$

Continued equation (53)

$$\left. \begin{aligned} C_1 &= -\frac{1}{2\pi\Delta_2} \left[ \begin{aligned} &(\bar{\gamma}_1 + 1)(\bar{\gamma}_2 + 1)e^{\lambda(2h_1+2h_2-x'_3 \sin \theta)} + \\ &(\bar{\gamma}_1 - 1)(\bar{\gamma}_2 - 1)e^{\lambda(4h_1-x'_3 \sin \theta)} + \\ &(\bar{\gamma}_1 + 1)(\bar{\gamma}_2 - 1)e^{\lambda(2h_1+x'_3 \sin \theta)} + \\ &(\bar{\gamma}_1 - 1)(\bar{\gamma}_2 + 1)e^{\lambda(2h_2+x'_3 \sin \theta)} \end{aligned} \right] \\ D_1 &= -\frac{1}{2\pi\Delta_2} \left\{ \begin{aligned} &(\bar{\gamma}_1 + 1)(\bar{\gamma}_2 - 1) [e^{\lambda(2h_1+x'_3 \sin \theta)} - e^{\lambda(2h_1-x'_3 \sin \theta)}] + \\ &(\bar{\gamma}_1 - 1)(\bar{\gamma}_2 + 1) [e^{\lambda(2h_2+x'_3 \sin \theta)} - e^{\lambda(2h_2-x'_3 \sin \theta)}] \end{aligned} \right\} \\ C_2 &= \frac{(\bar{\gamma}_2 + 1)}{\pi\Delta_2} [e^{\lambda(2h_1+2h_2+x'_3 \sin \theta)} - e^{\lambda(2h_1+2h_2-x'_3 \sin \theta)}] \\ D_2 &= \frac{(\bar{\gamma}_2 - 1)}{\pi\Delta_2} [e^{\lambda(2h_1-x'_3 \sin \theta)} - e^{\lambda(2h_1+x'_3 \sin \theta)}] \\ C_3 &= \frac{2}{\pi\Delta_2} [e^{\lambda(2h_1+2h_2+x'_3 \sin \theta)} - e^{\lambda(2h_1+2h_2-x'_3 \sin \theta)}] \end{aligned} \right\}$$

where

$$\Delta_2 = \frac{(\bar{\gamma}_2 - 1)e^{2\lambda h_1}[(\bar{\gamma}_1 + 1) + (\bar{\gamma}_1 - 1)e^{2\lambda h_1}] + (\bar{\gamma}_2 + 1)e^{2\lambda h_2}[(\bar{\gamma}_1 - 1) + (\bar{\gamma}_1 + 1)e^{2\lambda h_1}]}{2} \quad (54)$$

and  $\bar{\gamma}_1 = \frac{\bar{\mu}_2}{\mu_1}$ ,  $\bar{\gamma}_2 = \frac{\bar{\mu}_3}{\mu_2}$ .

Now

$$\left. \begin{aligned} & A_1(\lambda)e^{-\lambda y_3} + B_1(\lambda)e^{\lambda y_3} = \\ & -\frac{1}{2\pi\Delta_2} \left[ \begin{aligned} & (\bar{\gamma}_1 + 1)(\bar{\gamma}_2 + 1)e^{\lambda(2h_1+2h_2-x'_3 \sin \theta)} + \\ & (\bar{\gamma}_1 - 1)(\bar{\gamma}_2 - 1)e^{\lambda(4h_1-x'_3 \sin \theta)} - \\ & (\bar{\gamma}_1 + 1)(\bar{\gamma}_2 - 1)e^{\lambda(2h_1+x'_3 \sin \theta)} - \\ & (\bar{\gamma}_1 - 1)(\bar{\gamma}_2 + 1)e^{\lambda(2h_2+x'_3 \sin \theta)} \end{aligned} \right] e^{-\lambda y_3} + \\ & \frac{1}{2\pi\Delta_2} \left\{ \begin{aligned} & (\bar{\gamma}_1 + 1)(\bar{\gamma}_2 - 1) \left[ e^{\lambda(2h_1+x'_3 \sin \theta)} + e^{\lambda(2h_1-x'_3 \sin \theta)} \right] + \\ & (\bar{\gamma}_1 - 1)(\bar{\gamma}_2 + 1) \left[ e^{\lambda(2h_2+x'_3 \sin \theta)} + e^{\lambda(2h_2-x'_3 \sin \theta)} \right] \end{aligned} \right\} e^{\lambda y_3} \end{aligned} \right\} \quad (55)$$

First part of (55)

$$= -\frac{1}{2\pi} \left[ \begin{aligned} & \frac{(\bar{\gamma}_1 + 1)(\bar{\gamma}_2 + 1)}{\Delta_2} e^{\lambda(2h_1+2h_2-x'_3 \sin \theta - y_3)} + \\ & \frac{(\bar{\gamma}_1 - 1)(\bar{\gamma}_2 - 1)}{\Delta_2} e^{\lambda(4h_1-x'_3 \sin \theta - y_3)} - \\ & \frac{(\bar{\gamma}_1 + 1)(\bar{\gamma}_2 - 1)}{\Delta_2} e^{\lambda(2h_1+x'_3 \sin \theta - y_3)} - \\ & \frac{(\bar{\gamma}_1 - 1)(\bar{\gamma}_2 + 1)}{\Delta_2} e^{\lambda(2h_2+x'_3 \sin \theta - y_3)} \end{aligned} \right]$$

Now second part of (55)

$$\frac{1}{2\pi} \left\{ \begin{aligned} & \frac{(\bar{\gamma}_1 + 1)(\bar{\gamma}_2 - 1)}{\Delta_2} \left[ e^{\lambda(2h_1+x'_3 \sin \theta + y_3)} + e^{\lambda(2h_1-x'_3 \sin \theta + y_3)} \right] + \\ & \frac{(\bar{\gamma}_1 - 1)(\bar{\gamma}_2 + 1)}{\Delta_2} \left[ e^{\lambda(2h_2+x'_3 \sin \theta + y_3)} + e^{\lambda(2h_2-x'_3 \sin \theta + y_3)} \right] \end{aligned} \right\}$$

Now

$$\left. \begin{aligned} & \frac{(\bar{\gamma}_1 + 1)(\bar{\gamma}_2 + 1)}{\Delta_2} = \frac{e^{-2\lambda(h_1+h_2)}}{M} \\ & \frac{(\bar{\gamma}_1 - 1)(\bar{\gamma}_2 - 1)}{\Delta_2} = \frac{\bar{a}_1 \bar{c}_1 e^{-2\lambda(h_1+h_2)}}{M} \\ & \frac{(\bar{\gamma}_1 + 1)(\bar{\gamma}_2 - 1)}{\Delta_2} = \frac{\bar{a}_1 e^{-2\lambda(h_1+h_2)}}{M} \\ & \frac{(\bar{\gamma}_1 - 1)(\bar{\gamma}_2 + 1)}{\Delta_2} = \frac{\bar{c}_1 e^{-2\lambda(h_1+h_2)}}{M} \end{aligned} \right\}$$

Where  $M = 1 + \bar{a}_1 e^{-2\lambda h_2} + \bar{a}_1 \bar{c}_1 e^{-2\lambda(h_2-h_1)} + \bar{c}_1 e^{-2\lambda h_1}$ ,  $\bar{a}_1 = \frac{\bar{\gamma}_2-1}{\bar{\gamma}_2+1}$  and  $\bar{c}_1 = \frac{\bar{\gamma}_1-1}{\bar{\gamma}_1+1}$ .

Now the term  $|\bar{a}_1 e^{-2\lambda h_2} + \bar{a}_1 \bar{c}_1 e^{-2\lambda(h_2-h_1)} + \bar{c}_1 e^{-2\lambda h_1}| < 1$  (Mondal and Sen [13]) and we can express  $M$  as an infinite geometric series and neglecting the higher order term and we get

$$A_1(\lambda)e^{-\lambda y_3} + B_1(\lambda)e^{\lambda y_3} = \frac{1}{2\pi M} \left[ \begin{aligned} & - \left\{ \begin{aligned} & e^{-\lambda(x'_3 \sin \theta + y_3)} + \bar{a}_1 \bar{c}_1 e^{-\lambda(2h_2-2h_1+x'_3 \sin \theta + y_3)} - \\ & \bar{a}_1 e^{-\lambda(2h_2-x'_3 \sin \theta + y_3)} - \bar{c}_1 e^{-\lambda(2h_1-x'_3 \sin \theta + y_3)} \end{aligned} \right\} + \\ & \left\{ \begin{aligned} & \bar{a}_1 e^{-\lambda(2h_2-x'_3 \sin \theta - y_3)} + \bar{a}_1 e^{-\lambda(2h_2+x'_3 \sin \theta - y_3)} + \\ & \bar{c}_1 e^{-\lambda(2h_1-x'_3 \sin \theta - y_3)} + \bar{c}_1 e^{-\lambda(2h_1+x'_3 \sin \theta - y_3)} \end{aligned} \right\} \end{aligned} \right] \quad (56)$$

Expanding  $M$  as in infinite Geometric series and putting the value of  $A_1(\lambda)e^{-\lambda y_3} + B_1(\lambda)e^{\lambda y_3}$  in the equation (46) and integrating we get

$$G_{12(1)}^1 = \left. \begin{aligned} & \frac{1}{2\pi} \left[ -\frac{d_1}{d^2+d_1^2} + \frac{\bar{a}_1 \bar{c}_1 d_1}{(d-2h_1+2h_2)^2+d_1^2} + \frac{\bar{c}_1 d_1}{(d+2h_1)^2+d_1^2} + \right. \\ & \frac{\bar{a}_1 d_1}{(d+2h_2)^2+d_1^2} - \frac{\bar{a}_1 \bar{c}_1 d_1}{(d-2h_1+2h_2)^2+d_1^2} + \frac{\bar{a}_1^2 \bar{c}_1 d_1}{(d-2h_1+4h_2)^2+d_1^2} + \\ & \frac{\bar{a}_1^2 \bar{c}_1^2 d_1}{(d+4h_2-4h_1)^2+d_1^2} + \frac{\bar{a}_1 \bar{c}_1^2 d_1}{(d+2h_2)^2+d_1^2} + \frac{\bar{a}_1 d_1}{(d_2+2h_2)^2+d_1^2} - \\ & \frac{\bar{a}_1^2 d_1}{(d_2+4h_2)^2+d_1^2} - \frac{\bar{a}_1^2 \bar{c}_1 d_1}{(d_2-2h_1+4h_2)^2+d_1^2} - \frac{\bar{a}_1 \bar{c}_1 d_1}{(d_2+2h_1+2h_2)^2+d_1^2} + \\ & \frac{\bar{c}_1 d_1}{(d_2+2h_1)^2+d_1^2} - \frac{\bar{a}_1 \bar{c}_1 d_1}{(d_2+2h_1+2h_2)^2+d_1^2} - \frac{\bar{c}_1^2 \bar{a}_1 d_1}{(d_2+2h_2)^2+d_1^2} - \\ & \frac{\bar{c}_1^2 d_1}{(d_2+4h_1)^2+d_1^2} + \frac{\bar{a}_1 d_1}{(2h_2-d)^2+d_1^2} - \frac{\bar{a}_1^2 d_1}{(4h_2-d)^2+d_1^2} - \\ & \frac{\bar{a}_1^2 \bar{c}_1 d_1}{(4h_2-2h_1-d)^2+d_1^2} - \frac{\bar{a}_1 \bar{c}_1 d_1}{(2h_1+2h_2-d)^2+d_1^2} + \frac{\bar{a}_1 d_1}{(2h_2-d_2)^2+d_1^2} - \\ & \frac{\bar{a}_1^2 d_1}{(4h_2-d_2)^2+d_1^2} - \frac{\bar{a}_1^2 \bar{c}_1 d_1}{(4h_2-2h_1-d_2)^2+d_1^2} - \frac{\bar{a}_1 \bar{c}_1 d_1}{(2h_1+2h_2-d_2)^2+d_1^2} + \\ & \frac{\bar{c}_1 d_1}{(2h_1-d)^2+d_1^2} - \frac{\bar{a}_1 \bar{c}_1 d_1}{(2h_1+2h_2-d)^2+d_1^2} - \frac{\bar{a}_1 \bar{c}_1^2 d_1}{(2h_2-d)^2+d_1^2} - \\ & \frac{\bar{c}_1^2 d_1}{(4h_1-d)^2+d_1^2} + \frac{\bar{c}_1 d_1}{(2h_1-d_2)^2+d_1^2} - \frac{\bar{a}_1 \bar{c}_1 d_1}{(2h_2+2h_1-d_2)^2+d_1^2} - \\ & \left. \frac{\bar{a}_1 \bar{c}_1^2 d_1}{(2h_2-d_2)^2+d_1^2} - \frac{\bar{c}_1^2 d_1}{(4h_1-d_2)^2+d_1^2} - \frac{d_1}{d_2^2+d_1^2} \right] \end{aligned} \right\} \quad (57)$$

where  $d = x'_3 \sin \theta + y_3$ ,  $d_1 = x'_3 \cos \theta - y_2$ ,  $d_2 = -x'_3 \sin \theta + y_3$ .

Similarly we can calculate the terms  $G_{12(2)}^1$ ,  $G_{12(3)}^1$ ,  $G_{13(1)}^1$ ,  $G_{13(2)}^1$ ,  $G_{13(3)}^1$  and putting these values of  $G_{12(1)}^1, \dots, G_{13(3)}^1$  in the equations (43)-(45) and we get

$$\left. \begin{aligned} (\bar{u}_1)_2(Q_1) &= \frac{U}{2\pi} \bar{\psi}_1(y_2, y_3, p) \\ (\bar{u}'_1)_2(Q_2) &= \frac{U}{\pi} \bar{\phi}_1(y_2, y_3, p) \\ (\bar{u}''_1)_2(Q_3) &= \frac{2U}{\pi} \bar{\chi}_1(y_2, y_3, p) \end{aligned} \right\} \quad (58)$$

where

$$\begin{aligned}
 \bar{\psi}_1(y_2, y_3, p) = & \int_0^l \frac{f(x'_3)}{p} \left[ \frac{(y_2 \sin \theta + y_3 \cos \theta)}{A_{01}} - \frac{\bar{c}_1 \{y_2 \sin \theta + (y_3 + 2h_1) \cos \theta\}}{A_{02}} - \right. \\
 & \frac{\bar{a}_1 \{y_2 \sin \theta + (y_3 + 2h_2) \cos \theta\}}{A_{03}} - \frac{\bar{a}_1^2 \bar{c}_1 \{y_2 \sin \theta + (y_3 + 4h_2 - 2h_1) \cos \theta\}}{A_{04}} - \\
 & \frac{\bar{a}_1^2 \bar{c}_1^2 \{y_2 \sin \theta + (y_3 + 4h_2 - 4h_1) \cos \theta\}}{A_{05}} - \frac{\bar{a}_1 \bar{c}_1^2 \{y_2 \sin \theta + (y_3 + 2h_2) \cos \theta\}}{A_{03}} + \\
 & \frac{\bar{a}_1 \{-y_2 \sin \theta + (y_3 + 2h_2) \cos \theta\}}{A_{06}} - \frac{\bar{a}_1^2 \{-y_2 \sin \theta + (y_3 + 4h_2) \cos \theta\}}{A_{07}} - \\
 & \frac{\bar{a}_1^2 \bar{c}_1 \{-y_2 \sin \theta + (y_3 + 4h_2 - 2h_1) \cos \theta\}}{A_{08}} - \\
 & \frac{\bar{a}_1 \bar{c}_1 \{-y_2 \sin \theta + (y_3 + 2h_2 + 2h_1) \cos \theta\}}{A_{09}} + \\
 & \frac{\bar{c}_1 \{-y_2 \sin \theta + (y_3 + 4h_1) \cos \theta\}}{A_{10}} - \frac{\bar{a}_1 \bar{c}_1 \{-y_2 \sin \theta + (y_3 + 2h_2 + 2h_1) \cos \theta\}}{A_{09}} \\
 & \frac{\bar{a}_1 \bar{c}_1^2 \{-y_2 \sin \theta + y_3 \cos \theta\}}{A_{06}} - \frac{\bar{c}_1^2 \{-y_2 \sin \theta + (y_3 + 4h_1) \cos \theta\}}{A_{11}} + \\
 & \frac{\bar{a}_1 \{-y_2 \sin \theta + (2h_2 - y_3) \cos \theta\}}{A_{12}} - \frac{\bar{a}_1^2 \{-y_2 \sin \theta + (4h_2 - y_3) \cos \theta\}}{A_{13}} - \\
 & \frac{\bar{a}_1^2 \bar{c}_1 \{-y_2 \sin \theta + (4h_2 - 2h_1 - y_3) \cos \theta\}}{A_{14}} - \\
 & \frac{\bar{a}_1 \bar{c}_1 \{-y_2 \sin \theta + (2h_2 + 2h_1 - y_3) \cos \theta\}}{A_{15}} + \frac{\bar{a}_1 \{(x'_3 \cos \theta - y_2) \sin \theta\}}{A_{16}} - \\
 & \frac{\bar{a}_1^2 \{(x'_3 \cos \theta - y_2) \sin \theta\}}{A_{17}} - \frac{\bar{a}_1^2 \bar{c}_1 \{(x'_3 \cos \theta - y_2) \sin \theta\}}{A_{18}} - \\
 & \frac{\bar{a}_1 \bar{c}_1 \{-y_2 \sin \theta + (2h_2 + 2h_1 - y_3) \cos \theta\}}{A_{15}} + \\
 & \frac{\bar{c}_1 \{(x'_3 \cos \theta - y_2) \sin \theta\}}{A_{20}} - \frac{\bar{a}_1 \bar{c}_1 \{-y_2 \sin \theta - (2h_2 + 2h_1 - y_3) \cos \theta\}}{A_{19}} - \\
 & \frac{\bar{a}_1 \bar{c}_1^2 \{-y_2 \sin \theta + (2h_2 - y_3) \cos \theta\}}{A_{12}} - \frac{\bar{c}_1^2 \{-y_2 \sin \theta + (4h_1 - y_3) \cos \theta\}}{A_{21}} + \\
 & \frac{\bar{c}_1 \{-y_2 \sin \theta - (2h_1 - y_3) \cos \theta\}}{A_{22}} - \frac{\bar{a}_1 \bar{c}_1 \{(x'_3 \cos \theta - y_2) \sin \theta\}}{A_{19}} - \\
 & \frac{\bar{a}_1 \bar{c}_1^2 \{-y_2 \sin \theta - (2h_2 - y_3) \cos \theta\}}{A_{23}} - \frac{\bar{c}_1^2 \{-y_2 \sin \theta - (4h_1 - y_3) \cos \theta\}}{A_{24}} - \\
 & \frac{\{-y_2 \sin \theta + y_3 \cos \theta\}}{A_{25}} - \frac{\bar{a}_1 \{(2h_2 - y_3 + x'_3 \sin \theta) \cos \theta\}}{A_{23}} + \\
 & \frac{\bar{a}_1^2 \{(4h_2 - y_3 + x'_3 \sin \theta) \cos \theta\}}{A_{26}} + \frac{\bar{a}_1^2 \bar{c}_1 \{(4h_2 - 2h_1 - y_3 + x'_3 \sin \theta) \cos \theta\}}{A_{27}} + \\
 & \left. \frac{\bar{a}_1 \bar{c}_1 \{(2h_1 + 2h_2 - y_3 + x'_3 \sin \theta) \cos \theta\}}{A_{19}} + \frac{\bar{c}_1 \{(2h_1 - y_3 - x'_3 \sin \theta) \cos \theta\}}{A_{28}} \right] dx'_3 \quad (59)
 \end{aligned}$$

$$\bar{\phi}_1(y_2, y_3, p) = \int_0^l f(x'_3) [(x'_3 \cos \theta - y_2) \sin \theta \left\{ -\frac{1}{p(\bar{\nu}_1 + 1)} \times \right. \\ \left. \left( \frac{1}{B_{01}} + \frac{1}{B_{02}} - \frac{1}{B_{03}} - \frac{1}{B_{04}} \right) + \frac{\bar{a}_1}{p(\bar{\nu}_1 + 1)} \left( \frac{1}{B_{05}} + \frac{1}{B_{06}} - \frac{1}{B_{07}} - \frac{1}{B_{08}} \right) + \right. \\ \left. \frac{\bar{a}_1 \bar{c}_1}{p(\bar{\nu}_1 + 1)} \left( \frac{1}{B_{09}} + \frac{1}{B_{10}} - \frac{1}{B_{07}} - \frac{1}{B_{08}} \right) + \frac{\bar{c}_1}{p(\bar{\nu}_1 + 1)} \left( \frac{1}{B_{11}} + \frac{1}{B_{12}} - \frac{1}{B_{13}} - \frac{1}{B_{14}} \right) \right\} - \\ \frac{1}{p(\bar{\nu}_1 + 1)} \left\{ \frac{(y_3 - x'_3 \sin \theta) \cos \theta}{B_{01}} - \frac{(y_3 + x'_3 \sin \theta) \cos \theta}{B_{02}} \right\} - \\ \frac{\bar{a}_1}{p(\bar{\nu}_1 + 1)} \left\{ \frac{(2h_2 - y_3 + x'_3 \sin \theta) \cos \theta}{B_{04}} - \frac{(2h_2 - y_3 - x'_3 \sin \theta) \cos \theta}{B_{03}} - \right. \\ \left. \frac{(2h_2 + y_3 - x'_3 \sin \theta) \cos \theta}{B_{05}} + \frac{(2h_2 + y_3 + x'_3 \sin \theta) \cos \theta}{B_{06}} \right\} + \\ \frac{\bar{a}_1^2}{p(\bar{\nu}_1 + 1)} \left\{ \frac{(4h_2 - y_3 + x'_3 \sin \theta) \cos \theta}{B_{08}} - \frac{(4h_2 - y_3 - x'_3 \sin \theta) \cos \theta}{B_{07}} \right\} + \\ \frac{2\bar{c}_1}{p(\bar{\nu}_1 + 1)} \left\{ \frac{(2h_1 + y_3 - x'_3 \sin \theta) \cos \theta}{B_{11}} - \frac{(2h_1 + y_3 + x'_3 \sin \theta) \cos \theta}{B_{12}} \right\} + \\ \frac{\bar{a}_1 \bar{c}_1}{p(\bar{\nu}_1 + 1)} \left\{ \frac{(2h_2 - 2h_1 + y_3 - x'_3 \sin \theta) \cos \theta}{B_{09}} - \frac{(2h_2 - 2h_1 + y_3 + x'_3 \sin \theta) \cos \theta}{B_{10}} + \right. \\ \left. \frac{(2h_2 + 2h_1 - y_3 + x'_3 \sin \theta) \cos \theta}{B_{14}} - \frac{(2h_2 + 2h_1 - y_3 - x'_3 \sin \theta) \cos \theta}{B_{13}} \right\} + \\ \left. \frac{\bar{a}_1^2 \bar{c}_1}{p(\bar{\nu}_1 + 1)} \left\{ \frac{(4h_2 - 2h_1 - y_3 + x'_3 \sin \theta) \cos \theta}{B_{15}} - \frac{(4h_2 - 2h_1 - y_3 - x'_3 \sin \theta) \cos \theta}{B_{10}} \right\} \right] dx'_3 \quad (60)$$

and

$$\bar{\chi}_1(y_2, y_3, p) = \int_0^l f(x'_3) [(x'_3 \cos \theta - y_2) \sin \theta \left\{ -\frac{1}{p(\bar{\nu}_1 + 1)(\bar{\nu}_2 + 1)} \left( \frac{1}{B_{01}} + \frac{1}{B_{02}} \right) \right. \\ \left. + \frac{\bar{a}_1}{p(\bar{\nu}_1 + 1)(\bar{\nu}_2 + 1)} \left( \frac{1}{B_{05}} + \frac{1}{B_{06}} \right) + \frac{\bar{a}_1 \bar{c}_1}{p(\bar{\nu}_1 + 1)(\bar{\nu}_2 + 1)} \left( \frac{1}{B_{09}} + \frac{1}{B_{10}} \right) + \right. \\ \left. \frac{\bar{c}_1}{p(\bar{\nu}_1 + 1)(\bar{\nu}_2 + 1)} \left( \frac{1}{B_{11}} + \frac{1}{B_{12}} \right) \right\} - \\ \frac{1}{p(\bar{\nu}_1 + 1)(\bar{\nu}_2 + 1)} \left\{ \frac{(y_3 - x'_3 \sin \theta) \cos \theta}{B_{01}} - \frac{(y_3 + x'_3 \sin \theta) \cos \theta}{B_{02}} \right\} + \\ \frac{\bar{a}_1}{p(\bar{\nu}_1 + 1)(\bar{\nu}_2 + 1)} \left\{ \frac{(2h_2 + y_3 - x'_3 \sin \theta) \cos \theta}{B_{05}} - \frac{(2h_2 + y_3 + x'_3 \sin \theta) \cos \theta}{B_{06}} \right\} + \\ \frac{\bar{a}_1 \bar{c}_1}{p(\bar{\nu}_1 + 1)(\bar{\nu}_2 + 1)} \left\{ \frac{(2h_2 - 2h_1 + y_3 - x'_3 \sin \theta) \cos \theta}{B_{09}} - \frac{(2h_2 - 2h_1 + y_3 + x'_3 \sin \theta) \cos \theta}{B_{10}} \right\} + \\ \left. \frac{\bar{c}_1}{p(\bar{\nu}_1 + 1)(\bar{\nu}_2 + 1)} \left\{ \frac{(2h_1 + y_3 - x'_3 \sin \theta) \cos \theta}{B_{11}} - \frac{(2h_1 + y_3 + x'_3 \sin \theta) \cos \theta}{B_{12}} \right\} \right] dx'_3 \quad (61)$$

Taking Laplace inversions of (58) we get

$$\left. \begin{aligned} (u_1)_2(Q_1) &= \frac{U}{2\pi} \psi_1(y_2, y_3, t) \\ (u'_1)_2(Q_2) &= \frac{U}{2\pi} \phi_1(y_2, y_3, t) \\ (u''_1)_2(Q_3) &= \frac{2U}{\pi} \chi_1(y_2, y_3, t) \end{aligned} \right\} (62)$$

where  $\psi_1(y_2, y_3, t)$ ,  $\phi_1(y_2, y_3, t)$  and  $\chi_1(y_2, y_3, t)$  are the Laplace inversions of  $\bar{\psi}_1(y_2, y_3, p)$ ,  $\bar{\phi}_1(y_2, y_3, p)$  and  $\bar{\chi}_1(y_2, y_3, p)$  respectively.

Now

$$\left. \begin{aligned} (\bar{\tau}_{12})_2 &= \frac{\mu_1 U}{2\pi} \frac{\partial}{\partial y_2} \bar{\psi}_1(y_2, y_3, p) \\ (\bar{\tau}_{13})_2 &= \frac{\mu_1 U}{2\pi} \frac{\partial}{\partial y_3} \bar{\psi}_1(y_2, y_3, p) \end{aligned} \right\} (63)$$

$$\left. \begin{aligned} (\bar{\tau}'_{12})_2 &= \frac{\bar{\mu}_2 U}{\pi} \frac{\partial}{\partial y_2} \bar{\phi}_1(y_2, y_3, p) \\ &= \frac{U}{\pi} \bar{\phi}_2(y_2, y_3, p) \\ (\bar{\tau}'_{13})_2 &= \frac{\bar{\mu}_2 U}{\pi} \frac{\partial}{\partial y_3} \bar{\phi}_1(y_2, y_3, p) \\ &= \frac{U}{\pi} \bar{\phi}_3(y_2, y_3, p) \end{aligned} \right\} (64)$$

$$\left. \begin{aligned} (\bar{\tau}''_{12})_2 &= \frac{\bar{\mu}_3 2U}{\pi} \frac{\partial}{\partial y_2} \bar{\chi}_1(y_2, y_3, p) \\ &= \frac{2U}{\pi} \bar{\chi}_2(y_2, y_3, p) \\ (\bar{\tau}''_{13})_2 &= \frac{\bar{\mu}_3 2U}{\pi} \frac{\partial}{\partial y_3} \bar{\chi}_1(y_2, y_3, p) \\ &= \frac{2U}{\pi} \bar{\chi}_3(y_2, y_3, p) \end{aligned} \right\} (65)$$

Taking Laplace inversions in the equations (63)-(65) we get

$$\left. \begin{aligned} (\tau_{12})_2 &= \frac{\mu_1 U}{2\pi} \frac{\partial}{\partial y_2} \psi_1(y_2, y_3, t) \\ &= \frac{\mu_1 U}{2\pi} \psi_2(y_2, y_3, t) \\ (\tau_{13})_2 &= \frac{\mu_1 U}{2\pi} \frac{\partial}{\partial y_3} \psi_1(y_2, y_3, t) \\ &= \frac{\mu_1 U}{2\pi} \psi_3(y_2, y_3, t) \end{aligned} \right\} (66)$$

$$\left. \begin{aligned} (\tau'_{12})_2 &= \frac{U}{\pi} \phi_2(y_2, y_3, t) \\ (\tau'_{13})_2 &= \frac{U}{\pi} \phi_3(y_2, y_3, t) \end{aligned} \right\} (67)$$

$$\left. \begin{aligned} (\tau''_{12})_2 &= \frac{2U}{\pi} \chi_2(y_2, y_3, t) \\ (\tau''_{13})_2 &= \frac{2U}{\pi} \chi_3(y_2, y_3, t) \end{aligned} \right\} (68)$$

where the expressions for  $\psi_1(y_2, y_3, t)$ ,  $\psi_2(y_2, y_3, t)$ ,  $\psi_3(y_2, y_3, t)$ ;  $\phi_1(y_2, y_3, t)$ ,  $\phi_2(y_2, y_3, t)$ ,  $\phi_3(y_2, y_3, t)$ ;  $\chi_1(y_2, y_3, t)$ ,  $\chi_2(y_2, y_3, t)$ ,  $\chi_3(y_2, y_3, t)$  are obtained from  $\bar{\psi}_1(y_2, y_3, p)$ ,  $\bar{\psi}_2(y_2, y_3, p)$ ,  $\bar{\psi}_3(y_2, y_3, p)$ ;  $\bar{\phi}_1(y_2, y_3, p)$ ,  $\bar{\phi}_2(y_2, y_3, p)$ ,  $\bar{\phi}_3(y_2, y_3, p)$ ;  $\bar{\chi}_1(y_2, y_3, p)$ ,  $\bar{\chi}_2(y_2, y_3, p)$ ,  $\bar{\chi}_3(y_2, y_3, p)$  respectively by taking inverse Laplace transform.

Hence we get the complete solution given by (17).

Where

$$\left. \begin{aligned}
 A_{01} &= (x'_3 \sin \theta + y_3)^2 + (x'_3 \cos \theta - y_2)^2 \\
 A_{02} &= (2h_1 + x'_3 \sin \theta + y_3)^2 + (x'_3 \cos \theta - y_2)^2 \\
 A_{03} &= (2h_2 + x'_3 \sin \theta + y_3)^2 + (x'_3 \cos \theta - y_2)^2 \\
 A_{04} &= (4h_2 - 2h_1 + x'_3 \sin \theta + y_3)^2 + (x'_3 \cos \theta - y_2)^2 \\
 A_{05} &= (4h_2 - 4h_1 + x'_3 \sin \theta + y_3)^2 + (x'_3 \cos \theta - y_2)^2 \\
 A_{06} &= (2h_2 - x'_3 \sin \theta + y_3)^2 + (x'_3 \cos \theta - y_2)^2 \\
 A_{07} &= (4h_2 - x'_3 \sin \theta + y_3)^2 + (x'_3 \cos \theta - y_2)^2 \\
 A_{08} &= (4h_2 - 2h_1 - x'_3 \sin \theta + y_3)^2 + (x'_3 \cos \theta - y_2)^2 \\
 A_{09} &= (2h_2 + 2h_1 - x'_3 \sin \theta + y_3)^2 + (x'_3 \cos \theta - y_2)^2 \\
 A_{10} &= (2h_1 - x'_3 \sin \theta + y_3)^2 + (x'_3 \cos \theta - y_2)^2 \\
 A_{11} &= (4h_1 - x'_3 \sin \theta + y_3)^2 + (x'_3 \cos \theta - y_2)^2 \\
 A_{12} &= (2h_2 - x'_3 \sin \theta - y_3)^2 + (x'_3 \cos \theta - y_2)^2 \\
 A_{13} &= (4h_2 - x'_3 \sin \theta - y_3)^2 + (x'_3 \cos \theta - y_2)^2 \\
 A_{14} &= (4h_2 - 2h_1 - x'_3 \sin \theta - y_3)^2 + (x'_3 \cos \theta - y_2)^2 \\
 A_{15} &= (2h_2 + 2h_1 - x'_3 \sin \theta - y_3)^2 + (x'_3 \cos \theta - y_2)^2 \\
 A_{16} &= (2h_2 + x'_3 \sin \theta - y_3)^2 + (x'_3 \cos \theta - y_2)^2 \\
 A_{17} &= (4h_2 + x'_3 \sin \theta - y_3)^2 + (x'_3 \cos \theta - y_2)^2 \\
 A_{18} &= (4h_2 - 2h_1 - x'_3 \sin \theta - y_3)^2 + (x'_3 \cos \theta - y_2)^2 \\
 A_{19} &= (2h_2 + 2h_1 + x'_3 \sin \theta - y_3)^2 + (x'_3 \cos \theta - y_2)^2 \\
 A_{20} &= (2h_1 - x'_3 \sin \theta - y_3)^2 + (x'_3 \cos \theta - y_2)^2 \\
 A_{21} &= (4h_1 - x'_3 \sin \theta - y_3)^2 + (x'_3 \cos \theta - y_2)^2 \\
 A_{22} &= (2h_1 + x'_3 \sin \theta - y_3)^2 + (x'_3 \cos \theta - y_2)^2 \\
 A_{23} &= (2h_2 + x'_3 \sin \theta - y_3)^2 + (x'_3 \cos \theta - y_2)^2 \\
 A_{24} &= (4h_1 + x'_3 \sin \theta - y_3)^2 + (x'_3 \cos \theta - y_2)^2 \\
 A_{25} &= (x'_3 \sin \theta - y_3)^2 + (x'_3 \cos \theta - y_2)^2 \\
 A_{26} &= (4h_2 + x'_3 \sin \theta - y_3)^2 + (x'_3 \cos \theta - y_2)^2 \\
 A_{27} &= (4h_2 - 2h_1 + x'_3 \sin \theta - y_3)^2 + (x'_3 \cos \theta - y_2)^2 \\
 A_{28} &= (2h_1 - x'_3 \sin \theta - y_3)^2 + (x'_3 \cos \theta - y_2)^2
 \end{aligned} \right\} (69)$$

and

$$\left. \begin{aligned}
 B_{01} &= (y_3 - x'_3 \sin \theta)^2 + (x'_3 \cos \theta - y_2)^2 \\
 B_{02} &= (y_3 + x'_3 \sin \theta)^2 + (x'_3 \cos \theta - y_2)^2 \\
 B_{03} &= (2h_2 - y_3 - x'_3 \sin \theta)^2 + (x'_3 \cos \theta - y_2)^2 \\
 B_{04} &= (2h_2 - y_3 + x'_3 \sin \theta)^2 + (x'_3 \cos \theta - y_2)^2 \\
 B_{05} &= (2h_2 + y_3 - x'_3 \sin \theta)^2 + (x'_3 \cos \theta - y_2)^2 \\
 B_{06} &= (2h_2 + y_3 + x'_3 \sin \theta)^2 + (x'_3 \cos \theta - y_2)^2 \\
 B_{07} &= (4h_2 - y_3 - x'_3 \sin \theta)^2 + (x'_3 \cos \theta - y_2)^2 \\
 B_{08} &= (4h_2 - y_3 + x'_3 \sin \theta)^2 + (x'_3 \cos \theta - y_2)^2 \\
 B_{09} &= (2h_2 - 2h_1 + y_3 - x'_3 \sin \theta)^2 + (x'_3 \cos \theta - y_2)^2 \\
 B_{10} &= (2h_2 - 2h_1 + y_3 + x'_3 \sin \theta)^2 + (x'_3 \cos \theta - y_2)^2 \\
 B_{11} &= (2h_1 + y_3 - x'_3 \sin \theta)^2 + (x'_3 \cos \theta - y_2)^2 \\
 B_{12} &= (2h_1 + y_3 + x'_3 \sin \theta)^2 + (x'_3 \cos \theta - y_2)^2 \\
 B_{13} &= (2h_2 + 2h_1 - y_3 - x'_3 \sin \theta)^2 + (x'_3 \cos \theta - y_2)^2 \\
 B_{14} &= (2h_2 + 2h_1 - y_3 + x'_3 \sin \theta)^2 + (x'_3 \cos \theta - y_2)^2 \\
 B_{15} &= (4h_2 - 2h_1 - y_3 + x'_3 \sin \theta)^2 + (x'_3 \cos \theta - y_2)^2 \\
 B_{16} &= (4h_2 - 2h_1 - y_3 - x'_3 \sin \theta)^2 + (x'_3 \cos \theta - y_2)^2
 \end{aligned} \right\} (70)$$

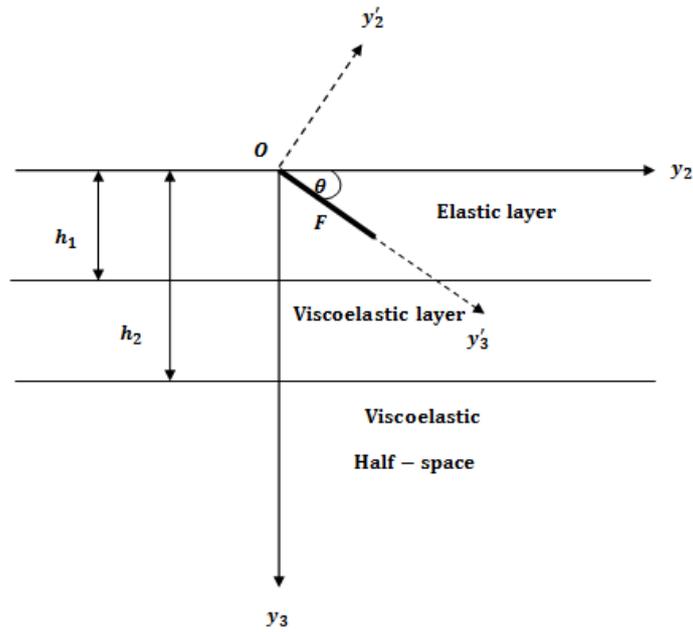


Fig. 1: Section of the model by the plane  $y_1 = 0$ .

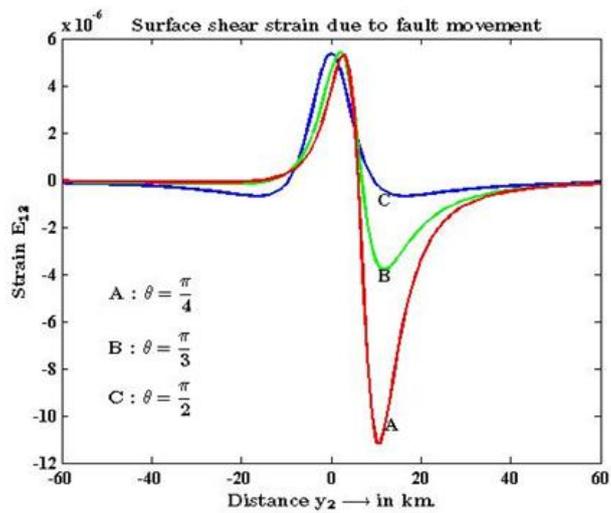


Fig. 2 Residual surface shear strain

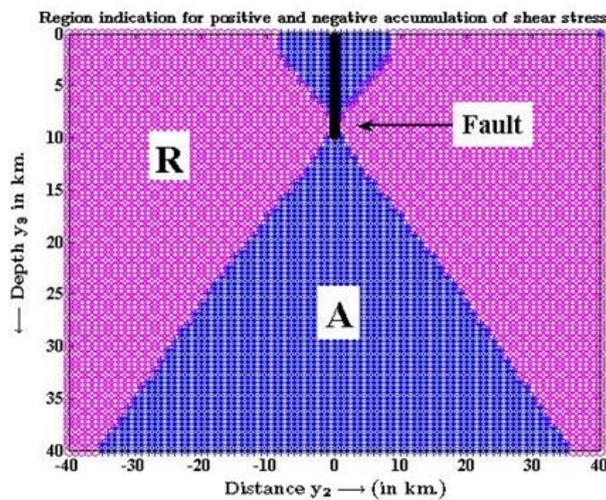
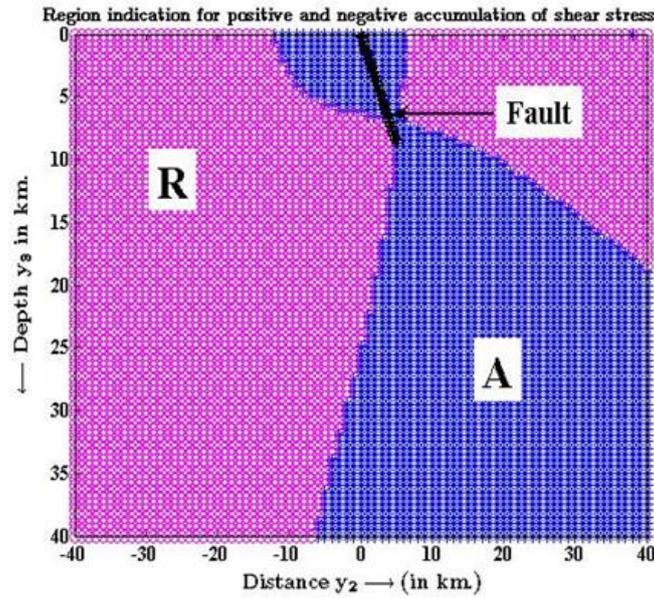
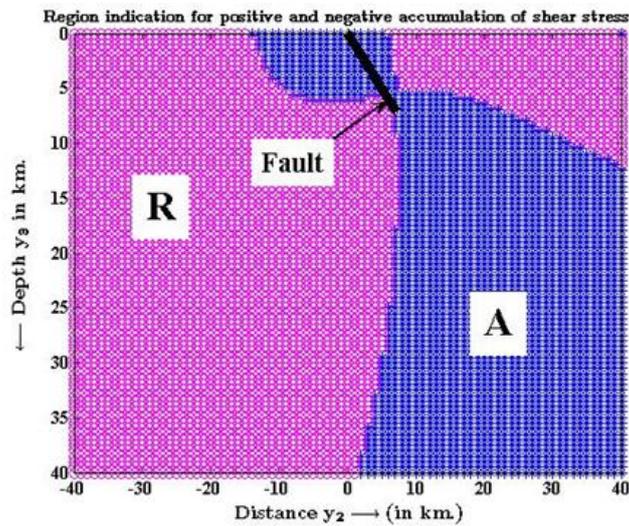


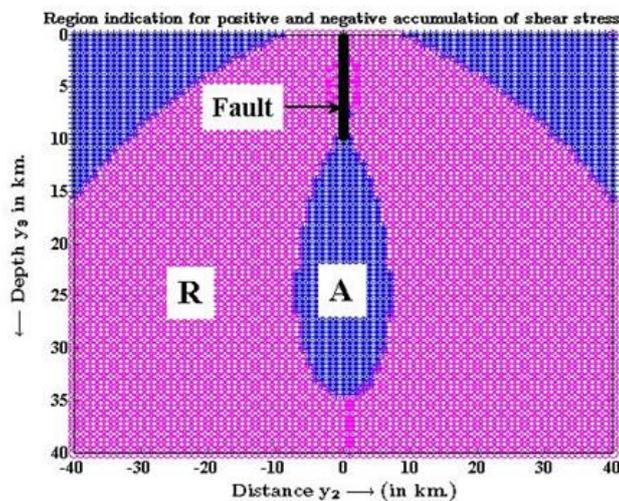
Fig. 3 Region of accumulation (A) and release (R) of shear stress in the first layer for  $\theta = \frac{\pi}{2}$ .



**Fig. 4** Region of accumulation (A) and release (R) of shear stress in the first layer for  $\theta = \frac{\pi}{3}$ .



**Fig. 5** Region of accumulation (A) and release (R) of shear stress in the first layer for  $\theta = \frac{\pi}{4}$ .



**Fig. 6** Region of accumulation (A) and release (R) of shear stress in the first layer for  $\theta = \frac{\pi}{2}$  and  $h_2 = 0$ .

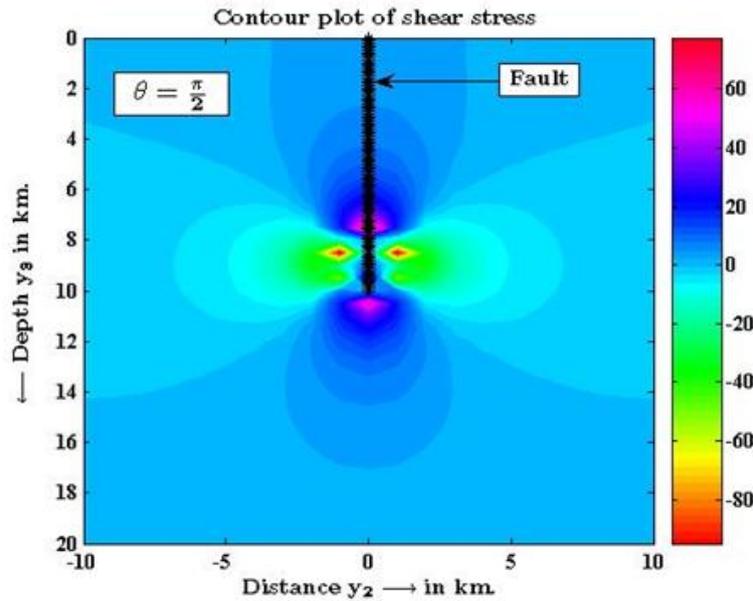


Fig. 7 Contour map of shear stress in the first layer for  $\theta = \frac{\pi}{2}$ .

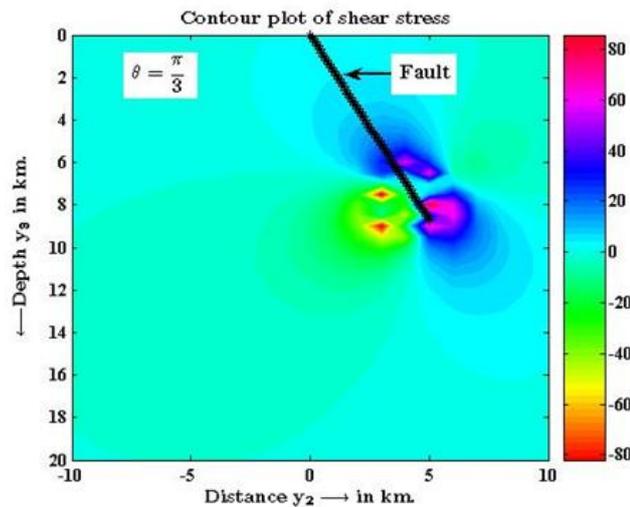


Fig. 7 Contour map of shear stress in the first layer for  $\theta = \frac{\pi}{3}$ .

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